

Beyond MSSM Baryogenesis

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Abstract

Taking the MSSM as an effective low-energy theory, with a cut-off scale of a few TeV, can make significant modifications to the predictions concerning the Higgs and stop sectors. We investigate the consequences of such a scenario for electroweak baryogenesis. We find that the window for MSSM baryogenesis is extended and, most important, can be made significantly more natural. Specifically, it is possible to have one stop lighter than the top and the other significantly lighter than TeV simultaneously with the Higgs mass above the LEP bound. In addition, various aspects concerning CP violation are affected. Most notably, it is possible to have dynamical phases in the bubble walls at tree level, providing CP violating sources for Standard Model fermions.

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I. INTRODUCTION

One of the strongest tests of the minimal supersymmetric standard model (MSSM) comes from the fact that within this framework, the mass of the lightest Higgs is bounded from above. Actually, the experimental lower bound on the Higgs mass [1] violates the tree level bound and implies that, if the MSSM is indeed realized in nature, stop-related loop corrections to the Higgs mass play an important role. For these corrections to be significant, the stop sector is required to exhibit special features: at least one of the stop mass eigenstates should be rather heavy and/or left-right-stop mixing should be substantial.

It is conceivable that additional new physics, beyond the MSSM, plays a role in particle interactions not far above the TeV scale. Model independent analyses of such scenarios were taken up in, for example, Refs. [2, 3, 4, 5] and, more recently, by Dine, Seiberg and Thomas [DST] in Ref. [6]. DST extend the MSSM by adding non-renormalizable terms that depend on the MSSM fields and are subject to the symmetries of the MSSM. They find two such terms that could increase the Higgs mass comparably to what loop corrections with heavy or mixed stops can do. These works demonstrate the sensitivity of the mass spectrum of the MSSM Higgs sector to changes in the quartic coupling.

Both the Higgs sector and the stop sector play a role in yet another interesting aspect of the MSSM, and that is the cosmological scenario of MSSM electroweak baryogenesis (EWBG) [7]. In this scenario, the two problems of the standard model baryogenesis – the fact that, with a single Higgs and taking into account experimental constraints on its mass, the electroweak phase transition (EWPT) cannot be strongly first order, and the fact that CP violation from the Kobayashi-Maskawa phase is much too small – are fixed by supersymmetric particles and their interactions. In particular, a light stop is crucial in making the EWPT strongly first order [8, 9]. It is therefore interesting to ask how do the DST operators affect EWBG. In particular, since these operators allow – given a fixed Higgs mass – lighter stops, do they open up new regions in the MSSM parameter space for successful baryogenesis?

In this work we focus on the EWPT and show that the parameter space required for a strong EWPT does indeed change qualitatively by DST operators. We discuss the implications of these qualitative features on the naturalness of the required parameter space. The various issues of calculating the baryon asymmetry of the universe (BAU) in a given

particle physics model are a subject of active research (see, for example, [10, 11, 12, 13]). We do not carry out, at this stage, a detailed explicit calculation of the BAU compatible with a low-cutoff MSSM, but rather point out several modifications by which the mechanism responsible for the produced baryon asymmetry may differ in the low-cutoff scenario from the renormalizable MSSM case.

The narrowing EWBG-window in the MSSM parameter space has led many authors to study extensions of the SM and of the MSSM in this context. Some recent examples are [14, 15, 16, 17, 18]. In most of these works, new particle degrees of freedom which are added to the model modify the EWPT by actively participating in thermal processes. Some previous studies have also considered the effect of non-renormalizable corrections to the SM potential [19, 20, 21]. Our analysis differs from previous studies in that it considers the most general non-renormalizable correction to the MSSM scalar potential. We find that there is no need for the beyond-MSSM (BMSSM) physics to actively couple to the thermal plasma in order to get significant modifications to various aspects of the EWBG.

The plan of this paper goes as follows. In Section II we review the predictions of the MSSM for the lightest Higgs mass. In Section III we give the modifications to these predictions due to the DST operators. In Section IV we present the modifications to the finite-temperature one-loop scalar potential from the DST terms. Our main results are derived in Section V where we explain how the contributions of the non-renormalizable terms enlarge the window for MSSM baryogenesis and, in particular, relax the fine-tuning problem of this scenario. The effects of these terms on the CP violating aspects of MSSM baryogenesis are discussed in Section VI: first, the possibility of CP violating bubble wall profiles (Subsection VIA) and, second, the modification to neutralino and chargino currents (Subsection VIB). We summarize our results in Section VII. In Appendix A we clarify some subtleties related to substituting the dimensionful parameters of the Higgs potential with measurable quantities in the presence of the DST operators.

II. THE LIGHTEST HIGGS MASS IN THE MSSM

Within the MSSM, the lightest Higgs mass is bounded from above. We write

$$m_h^2 = m_h^{2(\text{tree})} + m_h^{2(\text{loop})}. \quad (1)$$

The tree level contribution is given by

$$m_h^{2(\text{tree})} = \frac{1}{2} \left[m_Z^2 + m_A^2 - \sqrt{(m_A^2 - m_Z^2)^2 + 4m_A^2 m_Z^2 \sin^2 2\beta} \right] \\ \approx m_Z^2 - \frac{4m_Z^2 m_A^2}{m_A^2 - m_Z^2} \cot^2 \beta, \quad (2)$$

where the second, approximate equality holds for large $\tan \beta$. The most significant loop contribution is given by

$$m_h^{2(\text{loop})} = \frac{3m_t^4}{4\pi^2 v^2} \left[\ln \left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right) + \frac{|X_t|^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \ln \left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} \right) \right. \\ \left. + \frac{1}{2} \left(\frac{|X_t|^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \right)^2 \left(2 - \frac{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \ln \left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} \right) \right) \right], \quad (3)$$

where $X_t = A_t - \mu^* \cot \beta$.

The current lower bound from LEP II is $m_{h_{\text{SM}}} \gtrsim 114$ GeV, well above the upper bound on the tree contribution, $m_h^{(\text{tree})} \leq m_Z$. This contribution by itself is maximized at moderate to large $\tan \beta$. The top-stop loop correction should be substantial. If stop mixing is small, $|X_t/m_{\tilde{t}_{1,2}}|^2 \ll 1$, the correction depends only on the logarithm of the stop masses, so these must be rather heavy:

$$(m_{\tilde{t}_1} m_{\tilde{t}_2})^{1/2} \sim m_t \exp \left[\frac{2\pi^2 v^2 (m_h^2 - m_Z^2)}{3m_t^4} \right] \\ \sim 500 \text{ GeV} \times \exp \left\{ 2.9 \left[\left(\frac{m_h}{114 \text{ GeV}} \right)^2 - 1 \right] \right\}. \quad (4)$$

If, however, stop mixing is large, much lighter stops can still yield large loop corrections.

III. THE DST OPERATORS AND THE LIGHT HIGGS MASS

DST consider the following two non-renormalizable terms [6]:

$$W_{\text{DST}} = \frac{\lambda_1}{M} (H_u H_d)^2 + \frac{\lambda_2}{M} \mathcal{Z} (H_u H_d)^2, \quad (5)$$

where \mathcal{Z} is a SUSY-breaking spurion:

$$\mathcal{Z} = \theta^2 m_{\text{susy}}. \quad (6)$$

The first term in Eq. (5) is supersymmetric, while the second breaks SUSY. In the scalar potential, the following quartic terms are generated:

$$\frac{2\mu^* \lambda_1}{M} H_u H_d (H_u^\dagger H_u + H_d^\dagger H_d) - \frac{m_{\text{susy}} \lambda_2}{M} (H_u H_d)^2. \quad (7)$$

We define

$$\epsilon_1 \equiv \frac{\mu^* \lambda_1}{M}, \quad \epsilon_2 \equiv -\frac{m_{\text{susy}} \lambda_2}{M}. \quad (8)$$

The two terms in Eq. (7) contribute to the lightest Higgs boson mass as follows:

$$\begin{aligned} m_h^{2(\text{dst})} &= 2v^2 \left[\epsilon_{2r} - 2\epsilon_{1r} \sin 2\beta - \frac{2\epsilon_{1r} \sin 2\beta (m_A^2 + m_Z^2) + \epsilon_{2r} \cos^2 2\beta (m_A^2 - m_Z^2)}{\sqrt{(m_A^2 - m_Z^2)^2 + 4m_A^2 m_Z^2 \sin^2 2\beta}} \right] \\ &\simeq -16v^2 \epsilon_{1r} \cot \beta \frac{m_A^2}{m_A^2 - m_Z^2} + \mathcal{O}(\epsilon_i \cot^2 \beta), \end{aligned} \quad (9)$$

where $\epsilon_{ir} = \mathcal{R}e(\epsilon_i)$. Eq. (9) gives the leading ϵ_i -related shift to m_h , as long as $\cot \beta \not\ll \epsilon_i$. (Otherwise, $\mathcal{O}(\epsilon_i^2)$ corrections are equally or even more important.)

DST give the following numerical example. Take $m_{\tilde{t}_{1,2}} \simeq 300 \text{ GeV}$ with small mixing, $X_t \simeq 0$. Then, at moderate to large $\tan \beta$, we obtain $m_h \simeq 100 \text{ GeV}$. In the same small mixing limit, and taking conservatively $m_A \gg m_Z$, the additional correction (9) can accommodate $m_h \gtrsim 114 \text{ GeV}$ for $\epsilon_{1r} \cot \beta \lesssim -0.006$ (*e.g.*, $\epsilon_{1r} = -0.06$ and $\tan \beta = 10$). Assume now that the right-handed stop mass is bounded from above: $m_{\tilde{t}_R} \lesssim 170 \text{ GeV}$ (the relevance of this bound to our purposes will become clear below). With $\epsilon_1 \cot \beta = -0.006$, one obtains for the left-handed stop $m_{\tilde{t}_L} \gtrsim 530 \text{ GeV}$. We learn that the heavy stop mass is pushed quite high. Setting, however, $\epsilon_1 = 0$, would force it to much higher values: $m_{\tilde{t}_L} \gtrsim 2 \text{ TeV}$. Any further increase in the Higgs mass would correspond to an exponential increase in the heavy stop mass, implying fine tuning.

IV. THE SCALAR POTENTIAL

The MSSM finite-temperature effective potential was calculated by several groups [8, 9, 22, 23, 24, 25, 26, 27, 28, 29, 30]. Two-loop results are significant in improving the one-loop calculations and, as demonstrated by non-perturbative analyses, provide a rather accurate description of the full result [31, 32]. The qualitative changes that follow from adding the DST operators to the MSSM potential can, however, be well understood without the two-loop improvement. In fact, the effect which we find most significant is a zero-temperature effect which leaves the thermal computation all but idle. In this work we therefore employ the one-loop analysis. We leave a detailed quantitative discussion of the modified MSSM parameter space to more sophisticated, two-loop computations.

We represent the Higgs fields in the following way:

$$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} = \begin{pmatrix} \phi_1 + \frac{H_{dr}^0 + iH_{di}^0}{\sqrt{2}} \\ H_d^- \end{pmatrix}, \quad H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} = \begin{pmatrix} H_u^+ \\ \phi_2 + \frac{H_{ur}^0 + iH_{ui}^0}{\sqrt{2}} \end{pmatrix} \quad (10)$$

with the VEVs $\phi_1 = \langle H_d^0 \rangle$ and $\phi_2 = \langle H_u^0 \rangle$. The DST terms, Eq. (7), contribute the following dimension-four (but *effective dimension*-five [6]) terms to the tree level effective potential:

$$V_{\text{dst}} = -2 (|\phi_1|^2 + |\phi_2|^2) [\epsilon_1 \phi_1 \phi_2 + \text{h.c.}] + [\epsilon_2 (\phi_1 \phi_2)^2 + \text{h.c.}] . \quad (11)$$

The dimension-six terms,

$$V_{\text{dst}}^{(6)} = |2\epsilon_1/\mu|^2 |\phi_1 \phi_2|^2 (|\phi_1|^2 + |\phi_2|^2) ,$$

are important if the quartic couplings by themselves destabilize the potential. This has been demonstrated for a SM-like Higgs [19, 20, 21]. We focus, however, on the case where the DST operators actually raise the quartic coupling above the level predicted by the MSSM. In this case, the dimension-six terms can be neglected if they are suppressed in comparison to the dimension-four terms:

$$\epsilon_1 |\phi/\mu|^2 \cot \beta \ll 1.$$

The dimension-six operators also enter the potential via field-dependent thermal corrections. In our case the leading high-temperature terms due to such corrections, which are absent otherwise, are of the form $\left| \frac{\epsilon_1}{\mu} \cot \beta \right|^2 (\phi^4 T^2 + \phi^2 T^4)$ (we omit here numerical factors). For $|\mu| \gtrsim 100$ GeV, and within the temperature range of interest, these contributions are suppressed in comparison with the usual $\phi^2 T^2$ thermal terms which are accompanied by much larger coefficients.

Including the DST terms of Eq. (11), the one-loop effective potential for the Higgs scalars at finite temperature is given by

$$\begin{aligned} V = & m_1^2 |\phi_1|^2 + m_2^2 |\phi_2|^2 - (m_{12}^2 \phi_1 \phi_2 + \text{h.c.}) + \frac{g^2 + g'^2}{8} (|\phi_1|^2 - |\phi_2|^2)^2 \\ & - 2 (|\phi_1|^2 + |\phi_2|^2) [\epsilon_1 \phi_1 \phi_2 + \text{h.c.}] + [\epsilon_2 (\phi_1 \phi_2)^2 + \text{h.c.}] \\ & + \sum_{i=\{\text{dof}\}} \frac{n_i m_i^4(\phi)}{64\pi^2} \left[\ln \left(\frac{m_i^2(\phi)}{Q^2} \right) - \frac{3}{2} \right] \\ & + \sum_{i=\{\text{dof}\}} n_i \frac{T^4}{2\pi^2} J_i \left(\frac{m_i^2(\phi)}{T^2} \right) + \sum_{i=\{\text{sca}\}} \frac{n_i T}{12\pi} [m_i^3(\phi) - \bar{m}_i^3(\phi, T)] . \end{aligned} \quad (12)$$

The first line corresponds to the tree-level MSSM terms. The second line corresponds to the tree-level DST contributions. The third line is the zero-temperature one-loop contribution. The summation goes over

$$\{\text{dof}\} = \{t, b, \tilde{t}_{1,2}, \tilde{b}_{1,2}, H_e, H_o, H_c, W_T, Z_T, \gamma_T, W_L, Z_L, \gamma_L\}, \quad (13)$$

with

$$\begin{aligned} n_t &= n_b = -12, \quad n_{\tilde{t}_{1,2}} = n_{\tilde{b}_{1,2}} = 6, \\ n_{H_e} &= n_{H_o} = 2, \quad n_{H_c} = 4, \\ n_{W_T} &= 4, \quad n_{Z_T} = n_{\gamma_T} = 2, \quad n_{W_L} = 2, \quad n_{Z_L} = n_{\gamma_L} = 1. \end{aligned} \quad (14)$$

Here H_e and H_o refer to, respectively, the two CP-even and two CP-odd neutral Higgs bosons; H_c are the charged Higgs bosons; sub-indices T and L stand for, respectively, transverse and longitudinal. The fourth line is the finite-temperature contribution. The J_i functions are defined by

$$J_i(r) = \int_0^\infty dx \, x^2 \ln[1 - (-1)^{2s_i} e^{-\sqrt{x^2+r}}]. \quad (15)$$

The last term corresponds to daisy improvement. The masses $\bar{m}_i^2(\phi, T)$ are the field- and temperature-dependent eigenvalues of the mass matrices with first-order thermal masses included. We use the conventions of Ref. [24], where the reader is referred to for further details. The summation is over

$$\{\text{sca}\} = \{\tilde{t}_{1,2}, \tilde{b}_{1,2}, H_e, H_o, H_c, W_L, Z_L, \gamma_L\}. \quad (16)$$

V. THE ELECTROWEAK PHASE TRANSITION

The most important effect that we find comes from the tree level change in the quartic couplings of the scalar potential of Eq. (12). The most significant consequence of this effect is that it allows a strongly first-order EWPT with a relatively light \tilde{t}_L . In this section, we analyze this effect.

The region in MSSM parameter space which is compatible with a strong enough first-order phase transition (the *MSSM window*) has two distinctive characteristics (see [33, 34, 35] and references therein):

1. A light, (mostly) right-handed stop:

$$m_{\tilde{t}_R} \lesssim m_t; \quad (17)$$

2. A light Higgs, close to the LEP lower bound:

$$m_h \approx 115 \text{ GeV}. \quad (18)$$

In order to understand how this window is affected by the DST operators, we now explain how these constraints come about.

The condition for the sphaleron processes in the broken phase not to erase the baryon asymmetry that is produced along the expanding bubble wall reads [36]

$$\frac{\sqrt{2}v_c}{T_c} \gtrsim 1. \quad (19)$$

Here $v_c = v(T_c)$ and T_c are the Higgs VEV and the temperature at the instance in which the symmetric and the asymmetric vacua become degenerate. The normalization is such that $v_0 = v(T = 0) = 174 \text{ GeV}$.

The light stop constraint, Eq. (17), comes from the need to reduce thermal screening for at least one scalar which has a large coupling to the Higgs field [8, 9]. EW precision measurements can be accommodated more easily if this light stop is dominantly ‘right-handed’. Let us focus on the case of large but finite $m_A^2 \gg m_Z^2$, relevant to the analysis of Ref. [6]. The minimization of the potential reduces in this case to a one dimensional problem [8, 37], yielding

$$\frac{v_c}{T_c} \approx \frac{E}{\lambda}. \quad (20)$$

Here E is the coefficient of the cubic (barrier) term, and λ is the effective quartic coupling for the light Higgs. If the soft mass-squared of \tilde{t}_R is chosen negative such that it cancels exactly the thermal mass at the critical temperature, one has

$$E \approx \frac{h_t^3 \sin^3 \beta (1 - X_t^2/m_Q^2)^{3/2}}{2\pi}. \quad (21)$$

For small stop mixing, $X_t^2/m_Q^2 \ll 1$, E can be of order 0.1 and thus an order of magnitude larger than the SM contribution due to transverse gauge bosons, $E_{\text{SM}} \sim 0.01$. Eq. (21) biases the MSSM window towards small stop mixing regions. Most importantly, the requirement of negative m_U^2 forces $m_{\tilde{t}_R} < m_t$. Within one-loop analysis, one must in fact impose a rather

strong constraint, $m_U^2 \sim -(80 \text{ GeV})^2$ or equivalently $m_{\tilde{t}_R} \sim 150 \text{ GeV}$, to obtain a strong enough PT.

Two-loop calculations (see, for example, [27, 33, 37] and references therein) extend the window by correcting Eq. (20):

$$\frac{v_c}{T_c} \approx \frac{E}{2\lambda} + \sqrt{\frac{E^2}{4\lambda^2} + \frac{c_2}{\lambda}}, \quad (22)$$

where c_2 is the coefficient of the generic two-loop correction,

$$\Delta V^{(2\text{-loop})} \approx -c_2 T^2 \phi^2 \ln \frac{\phi}{T}.$$

Eq. (22) explains how two-loop corrections make room for some stop mixing and relax the upper bound on m_U^2 . However, sizeable positive values of m_U^2 or large mixing are still forbidden, as they directly decrease E . In addition, the effective quartic coupling λ remains in the denominator of (22).

The light Higgs constraint, Eq. (18), can also be understood from the previous discussion. Since λ is proportional to the zero temperature value of the Higgs mass-squared (up to a weak logarithmic dependence which distinguishes λ of Eqs. (20,22) from the zero-temperature λ), $\lambda \approx m_h^2/(2v_0^2)$, the constraint of a light Higgs turns out to be a zero-temperature effect in the MSSM.

In fact, Refs. [26, 33] note that the requirement of a light Higgs in the MSSM window does not arise directly from the two-loop thermal calculation. Instead, it is a consequence of the theoretical “upper bound” $m_{\tilde{t}_L} \lesssim \text{TeV}$, coming from fine-tuning considerations. The question is how can an increase in the Higgs mass above the LEP bound be accounted for in the MSSM, where the stop-Higgs relation of Eq. (4) holds. Within the MSSM window, that is when Eq. (17) is obeyed, a corresponding (exponentially large!) increase of $m_{\tilde{t}_L}$ is required. In contrast, outside the baryogenesis window, the task can be shared among the two stops. Two-loop analyses [26, 33] show that this naive argument is qualitatively correct, though quantitatively the bound is somewhat weaker than what follows from Eq. (4). Within the MSSM window, they obtain

$$m_Q \cong 100 \text{ GeV} \times \exp[0.11 (m_h[\text{GeV}] - 85.9)]. \quad (23)$$

This strong constraint implies, for example, that in order to account for $m_h \sim 120 \text{ GeV}$, one must take $m_{\tilde{t}_L} \sim 4 \text{ TeV}$ within the MSSM window, while outside this window one can have $m_{\tilde{t}_L} \sim m_{\tilde{t}_R} \sim 680 \text{ GeV}$, a much less fine-tuned situation.

This is the point where the effect of the DST corrections (9) is most significant. The DST operators modify the quartic coupling at zero-temperature, and consequently the task of stabilizing the potential can be shared between them and the stop sector. This adds a new twist to a well-recognized fact: The smallness of the quartic coupling in the tree level MSSM makes the spectrum extremely sensitive not only to quantum corrections but also to non-renormalizable contributions. However, in contrast to the loop corrections which require in this case a strong fine tuning, the low-cutoff corrections relieve this tuning by allowing a light soft mass for the stop [38]. We demonstrate this effect in Fig. 1, where we plot curves of constant ϵ_1 in the $m_h - m_Q$ plane. For the purpose of illustration, we use vanishing stop mixing, $X_t = 0$, and a light right-handed stop, $m_{\tilde{t}_R} = 150 \text{ GeV}$, consistent with a strongly first-order EWPT at one-loop. One learns from this figure that even a modest non-renormalizable correction, $\epsilon_{1r} \sim -0.05$, suffices to reduce fine tuning in the Higgs sector by two orders of magnitude, making a sizable part of the traditional MSSM window significantly more natural.

In Fig. 2 we evaluate the one loop potential, in order to support our statement that the opening of the MSSM window for a lighter left-handed stop is a zero-temperature effect. We present in the figure the order parameter $v(T) = \sqrt{|\phi_1|^2 + |\phi_2|^2}$ at the true vacuum as a function of temperature for different values of ϵ . (Notice the first-order nature of the EWPT, even at one loop, for the selected set of parameters.) Keeping the left-handed stop heavy and decoupled, we see that the effect of varying ϵ is mainly to shift the resulting values of the critical VEV and temperature such that $(v_c/T_c) \cdot m_h^2 \approx \text{const}$. This confirms the simple expectation of Eq. (20), and shows that a change in v_c/T_c due to the DST operators is simply a result of the zero-temperature change in Higgs mass. Two-loop and lattice calculations then imply that a strongly first-order EWPT can occur even if m_h is well above the LEP bound.

Let us conclude this section by summarizing the qualitative picture that arises from our analysis. The main results can be understood from Eqs. (19) and (20). The resulting constraints on the relevant supersymmetric parameters can be schematically presented as follows:

$$\frac{E(m_{\tilde{t}_R})}{\lambda(m_{\tilde{t}_R}, m_{\tilde{t}_L}, \epsilon_i)} \geq 1. \quad (24)$$

Given that there is an *experimental* lower bound on λ , the requirement of strongly first order EWPT translates into a lower bound on E which, in our framework, requires $m_{\tilde{t}_R}$ to be within

the narrow range between the lower bound (coming from direct searches and/or the requirement that there is no color breaking) and (roughly) m_t . Since E does not depend directly on ϵ_i , this constraint is hardly affected by extending the MSSM with non-renormalizable terms. Thus, the main effect of ϵ_i is that it can be combined with $m_{\tilde{t}_L}$ to render λ close to the lower bound (with $m_{\tilde{t}_R}$ almost fixed). In particular, negative values of ϵ_1 allow lower values of $m_{\tilde{t}_L}$ compared to the MSSM. This is the content of Fig. 1. We expect other, smaller effects, on the allowed range of $\tan\beta$ and X_t , but to quantify them we need the full two-loop calculation.

VI. CP VIOLATION

The non-renormalizable terms affect not only the phase transition, but also the CP violation that is relevant to baryogenesis. First, they induce CP violating bubble wall profiles and by that allow the third generation fermions to directly produce some baryon asymmetry. Second, they modify the chargino and neutralino currents. These two effects are described in the two respective subsections.

A. CP violation in bubble wall

In the tree level potential of the MSSM, the m_1^2 and m_2^2 parameters are real, while the m_{12}^2 parameter can have an arbitrary phase. However, one may use global field redefinitions to make m_{12}^2 real and positive. In this basis, ϕ_1 and ϕ_2 are real and positive throughout the phase transition. Thus one arrives at the well known conclusion that, within the MSSM, CP violation in bubble walls is insignificant [39, 40, 41] (see, however, [42]).

Adding the DST operators changes this picture. Let us define

$$\begin{aligned} v^2 &= |\phi_1|^2 + |\phi_2|^2, \\ \tan\beta &= |\phi_1/\phi_2|, \quad s = \sin\beta, \quad c = \cos\beta, \\ e^{-i\theta} &= v^2 sc / (\phi_1 \phi_2). \end{aligned} \tag{25}$$

Note that v and β (and θ) parameterize the VEVs at the *finite*-temperature vacuum. The phase dependent part in the tree level potential is

$$- \left[(m_{12}^2 + 2v^2\epsilon_1) e^{-i\theta} - \epsilon_2 v^2 sc e^{-2i\theta} + \text{h.c.} \right] v^2 sc. \tag{26}$$

In contrast to the MSSM scenario, it is not possible – even at tree level – to globally define the phases of ϕ_i such that the coefficient in square brackets in Eq. (26) is maximized for varying v . During the PT, the phase of $\phi_1\phi_2$ aligns dynamically, inducing an energy gap between left and right chiral components of fermions [39]. This situation resembles the case of the two-Higgs doublet model [17]. It is different from the MSSM scenario in that, for example, the BAU can be generated through top or tau rather than chargino or neutralino currents.

The produced BAU due to a varying complex phase in the bubble wall is proportional to an integral over the gradient of the phase across the wall. To find the dynamical phase profile requires obtaining the tunneling solution, as in [41]. Here we use Eq. (26) to estimate the total variation of θ between the symmetric and broken vacua, based on potential energy alone. Let us neglect for now the effect of ϵ_2 . In this limit, θ is given by

$$\theta = \arg(m_{12}^2 + 2v^2\epsilon_1).$$

Adopting a basis for the symmetric vacuum in which m_{12}^2 is real and positive, we get, to leading order in ϵ_1 ,

$$\Delta\theta \approx 2\epsilon_{1i} \left(\frac{v_c}{m_A}\right)^2 \tan\beta \quad (27)$$

We substitute here $m_{12}^2 \approx \frac{1}{2}m_A^2 \sin 2\beta$, which holds at tree level and to zeroth order in ϵ (see Appendix A). Note that our approximations here and in Section III hold for $(m_A/v_0)^2 > 2\epsilon \tan\beta$. Thus the phase cannot be large, $\Delta\theta < 0.3$.

The tree level CP violation at the bubble walls triggers some amount of baryogenesis, through varying complex phases in the field-dependent masses of SM particles, like top and bottom quarks or tau leptons. Which of these plays the most significant role depends on the specifics of the scenario under consideration, particularly the value of $\tan\beta$.

B. SUSY particle currents

In the renormalizable MSSM, mass eigenmodes in the chargino and neutralino sectors develop time-varying complex phases due to the variation of EW breaking, real, off-diagonal terms in the associated mass matrices. CP violation is provided by complex-valued SUSY and soft SUSY-breaking parameters. DST operators affect this computation, even in case that they do not introduce any new phases.

We continue to use the parametrization of Eq. (25). The neutralino \tilde{N} and the chargino \tilde{C} mass matrices get contributions from the dimension-five DST operator [6]:

$$\frac{\epsilon_1}{\mu^*} \left[2(H_u H_d)(\tilde{H}_u \tilde{H}_d) + 2(\tilde{H}_u H_d)(H_u \tilde{H}_d) + (H_u \tilde{H}_d)(H_u \tilde{H}_d) + (\tilde{H}_u H_d)(\tilde{H}_u H_d) \right] + \text{h.c.}, \quad (28)$$

arising from the superpotential of Eq. (5). In the gauge eigenstate basis, one obtains [43]:

$$M_{\tilde{N}} = \begin{pmatrix} M_1 & 0 & -\frac{g'\phi_1}{\sqrt{2}} & \frac{g'\phi_2}{\sqrt{2}} \\ 0 & M_2 & \frac{g\phi_1}{\sqrt{2}} & -\frac{g\phi_2}{\sqrt{2}} \\ -\frac{g'\phi_1}{\sqrt{2}} & \frac{g\phi_1}{\sqrt{2}} & 0 & -\mu \\ \frac{g'\phi_2}{\sqrt{2}} & -\frac{g\phi_2}{\sqrt{2}} & -\mu & 0 \end{pmatrix} - \frac{\epsilon_1}{\mu^*} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_2^2 & 4\phi_1\phi_2 \\ 0 & 0 & 4\phi_1\phi_2 & \phi_1^2 \end{pmatrix}, \quad (29)$$

$$M_{\tilde{C}} = \begin{pmatrix} M_2 & g\phi_2 \\ g\phi_1 & \mu \end{pmatrix} + \frac{2\epsilon_1}{\mu^*} \phi_1\phi_2 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \quad (30)$$

Apart from modifications of the spectrum, the DST corrections introduce position dependence into the $\tilde{H} - \tilde{H}$ entries of the matrices. Position dependence of the diagonal elements is by itself a new effect. The gradients of DST-induced entries are, however, suppressed by factors of $\mathcal{O}(\epsilon\phi/\mu)$, small compared to the usual suppression of $\mathcal{O}(g)$.

VII. DISCUSSION

The tree-level renormalizable scalar potential of the MSSM yields an upper bound on the mass of the lightest Higgs boson which is experimentally known to be violated. Loop-corrections involving the top quarks and squarks can (still?) relax the theoretical bound sufficiently, at the cost of some fine-tuning. Non-renormalizable terms, suppressed by a cut-off scale in the few TeV range, can similarly relax the bound without, however, fine-tuning [6]. We analyzed the consequences of such beyond-MSSM (BMSSM) effects on electroweak baryogenesis (EWBG). We find that, even in this regime, the non-renormalizable terms may easily alter some of the principal features of the so called “window” for EWBG in the MSSM.

In particular, the fine-tuning that arises if the required increase of the quartic Higgs coupling is attributed solely to stop-related loop corrections, becomes quite strong in the MSSM window for baryogenesis. The reason is that, to have a large enough cubic term in the scalar potential as necessary for strongly first-order phase transition, the mass of the lighter

stop must be below the top mass which, in turn, requires that the heavy stop mass is in the few TeV range. In our framework, however, the task of increasing the quartic coupling can be shared between the stop-related loop-corrections and the non-renormalizable contributions. This allows a strongly first-order phase transition simultaneously with a Higgs mass above the LEP bound with the heavier stop mass well below TeV. Thus, the MSSM window is extended and, most significantly, made natural.

Additional relevant consequences of the non-renormalizable terms concern CP violation. The new operators provide new sources of CP violation. These make a qualitative change in the picture of CP violation. Unlike in the MSSM, it is impossible in general to choose a phase convention whereby the relative phase between the two Higgs VEVs vanishes all along the bubble wall. The interactions of the third generation fermions – top, bottom and tau – with the bubble wall could thus contribute significantly to the generation of the baryon asymmetry. Finally, the chargino and neutralino currents, which within the MSSM are usually responsible for the baryogenesis, are modified in a qualitatively interesting way, though the quantitative effects are parametrically suppressed and may turn out small.

We conclude that BMSSM effects, which may become necessary to give a consistent picture of the Higgs and stop sectors, can play a significant role also in supersymmetric baryogenesis. In particular, the BMSSM window for baryogenesis allows for parameters that are significantly more natural than those of the MSSM baryogenesis.

Acknowledgments

We are grateful to Michael Dine and Ann Nelson for useful discussions and to Michael Peskin, Scott Thomas and, in particular, Jose Espinosa, for helpful correspondence. This research is supported by the Israel Science Foundation founded by the Israel Academy of Sciences and Humanities, the United States-Israel Binational Science Foundation (BSF), Jerusalem, Israel, the German-Israeli foundation for scientific research and development (GIF), and the Minerva Foundation.

APPENDIX A: SUBSTITUTING m_1^2, m_2^2, m_{12}^2

The effective potential of Eq. (12) depends on three dimensionful quantities: m_1^2, m_2^2 and m_{12}^2 . These parameters need to be replaced by measurable quantities, for which we choose m_Z, m_A and $\tan \beta$ at the zero-temperature vacuum:

$$\begin{aligned}
m_{12}^2 &= \frac{1}{2}m_A^2 s_{2\beta} - 2\epsilon_{1r}v^2 + 2\epsilon_{2r}v^2 s_{2\beta} + \frac{3g^2 m_t^2 A_t \mu}{32\pi^2 s_\beta^2 m_W^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + \frac{3g^2 m_b^2 A_b \mu}{32\pi^2 c_\beta^2 m_W^2} g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2), \\
g(m_1^2, m_2^2) &= \frac{m_1^2 [\ln(m_1^2/Q^2) - 1] - m_2^2 [\ln(m_2^2/Q^2) - 1]}{m_1^2 - m_2^2}, \\
m_1^2 &= m_{12}^2 \tan \beta - \frac{1}{2}m_Z^2 c_{2\beta} + 2\epsilon_{1r}v^2 (\tan \beta + s_{2\beta}) - 2\epsilon_{2r}v^2 s_\beta^2 \\
&\quad - \frac{1}{64\pi^2} \left\{ \sum_{i=\{\text{dof}\}} n_i \frac{m_i^2(\phi)}{\phi_1} \frac{\partial m_i^2(\phi)}{\partial \phi_1} \left[\ln \left(\frac{m_i^2(\phi)}{Q^2} \right) - 1 \right] \right\}_{v,\beta}, \\
m_2^2 &= m_{12}^2 \cot \beta + \frac{1}{2}m_Z^2 c_{2\beta} + 2\epsilon_{1r}v^2 (\cot \beta + s_{2\beta}) - 2\epsilon_{2r}v^2 c_\beta^2 \\
&\quad - \frac{1}{64\pi^2} \left\{ \sum_{i=\{\text{dof}\}} n_i \frac{m_i^2(\phi)}{\phi_2} \frac{\partial m_i^2(\phi)}{\partial \phi_2} \left[\ln \left(\frac{m_i^2(\phi)}{Q^2} \right) - 1 \right] \right\}_{v,\beta}. \tag{A1}
\end{aligned}$$

The values of v, β in Eqs. (A1) refer to the zero-temperature vacuum.

The basis in which we define the phases of ϵ above is defined by having m_{12}^2 real and positive. Adopting the conventions of Eq. (25), the θ -dependent part in the tree-level potential is approximately given by

$$-2v^4 \cot \beta \cdot \mathcal{R}e \left[\left(\frac{m_{12}^2}{v^2} + 2\epsilon_1 - \cot \beta \epsilon_2 e^{-i\theta} \right) e^{-i\theta} \right]. \tag{A2}$$

For $m_{12}^2/v^2 \gg 2\epsilon$, there is a small θ solution which minimizes (A2):

$$\theta \approx \frac{v^2}{m_{12}^2} (2\epsilon_{1i} - \cot \beta \epsilon_{2i}),$$

to leading order in ϵ . For this solution, we can treat $\phi_{1,2}$ as positive numbers and replace the real part in (A2) by absolute value:

$$-2m_{12}^2 \phi_1 \phi_2 - 4\epsilon_{1r} (\phi_1^2 + \phi_2^2) \phi_1 \phi_2 + 2\epsilon_{2r} \phi_1^2 \phi_2^2,$$

which is used in Eqs. (A1).

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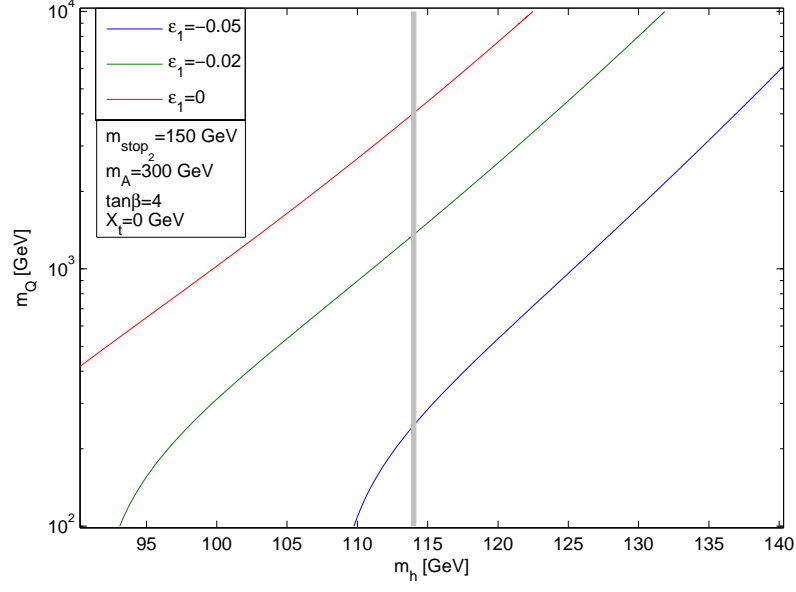


FIG. 1: The soft SUSY-breaking mass of \tilde{t}_L , m_Q , versus the light Higgs mass, m_h , for various values of ϵ_1 . Other relevant parameters are fixed at $X_t = 0$, $m_{\tilde{t}_R} = 150$ GeV, $m_A = 300$ GeV and $\tan \beta = 4$. The gray line marks the LEP lower bound, $m_h = 114$ GeV.

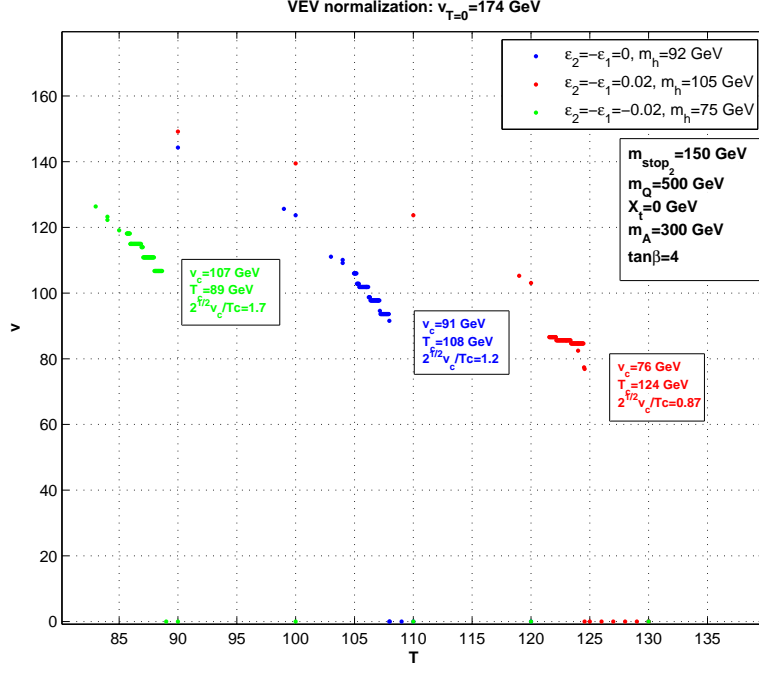


FIG. 2: The Higgs VEV at the lowest minimum as a function of temperature for various values of m_h , ϵ_1 and ϵ_2 . Other relevant parameters are fixed at $X_t = 0$, $m_{\tilde{t}_R} = 150$ GeV, $m_Q = 500$ GeV, $m_A = 300$ GeV and $\tan \beta = 4$. The different sets of $(m_h, \epsilon_1, \epsilon_2)$ all yield the same $m_h^2(v_c/T_c)$ to within about 5%.